

On-shell S-matrix and tachyonic effective actions

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ABSTRACT

We evaluate various disk level four-point functions involving the massless scalar and tachyon vertex operators in the presence of background B-flux in superstring theory. By studying these amplitudes in specific limits, we find couplings of two scalars with two tachyons, and couplings of four tachyons on the world-volume of non-BPS D-branes of superstring theory. They are fully consistent with the non-commutative tachyonic Dirac-Born-Infeld effective action. They also fix the coefficient of T^4 term in the expansion of the tachyon potential around its maximum.

1 The idea

Recently different tachyonic effective actions have been used to describe the time evolution of unstable D-branes in string theory[1, 2, 3, 4, 5]¹. In particular, Sen has shown that the string theory produces a pressure-less gas with non-zero energy density at the late time of the tachyon condensation[1]. In this paper, he showed that these results can be derived also from the tachyonic Dirac-Born-Infeld(DBI) effective action[7, 8] around its true vacume. Other possible applications of this action to cosmology have been discussed in [9]. This action was claimed in [7] that is deducible from on-shell string theory scattering amplitudes. In this paper we would like to see to what extent the string theory S-matrix elements produce this action.

In inferring effective action from string theory on-shell S-matrix elements, one usually evaluates the S-matrix elements and then expands it in the limit $\alpha' \rightarrow 0$, *i.e.*, low energy limit. Then one writes a low energy effective action in the field theory that reproduces the leading terms of the expansion. For the case that the field theory involves only massless fields, *i.e.*, flat potential, this expansion is unique, though many apparently different but physically identical low energy actions may produce them. They may in turn be related to each other by some field redefinition[10]. However, if one includes a tachyonic field, then the corresponding limit of the S-matrix elements is not $\alpha' \rightarrow 0$, in general, since the action around the maximum of the tachyon potential itself is not low energy effective action anymore. Different expansion of a S-matrix element may then correspond to physically different effective actions. For example, consider the S-matrix element of one closed string tachyon and two open string massless scalars, *i.e.*, flat potential, in Type 0 theory[11, 12]²,

$$A(\tau, X, X) \sim \zeta_1 \cdot \zeta_2 \frac{\Gamma(-2s)}{\Gamma(-s)\Gamma(-s)},$$

where $s = -2k_1 \cdot k_2$ and ζ^i, k^a are the scalar polarization and momentum, respectively³. The low energy expansion, *i.e.*, $s \rightarrow 0$, is

$$A \sim \zeta_1 \cdot \zeta_2 (k_1 \cdot k_2 + O(s^2)).$$

The first term above is reproduced in field theory by the low energy DBI action, and $O(s^2)$ terms are related to higher derivative terms which are not included in the DBI action.

Now consider the S-matrix element of one closed string tachyon and two open string tachyons at the top of the open string tachyon potential. A simple calculation yields

$$A(\tau, T, T) \sim \frac{\pi}{4} \frac{\Gamma(-2s)}{\Gamma(-s)\Gamma(-s)}, \quad (1)$$

¹For early studies of open string tachyon dynamics, see [6].

²For simplicity, we assumed that the closed string field does not depend on the transverse coordinates.

³Our convention sets $\alpha' = 2$. Our index conventions are that early Latin indices take values in the world-volume, *e.g.*, $a, b = 0, 1, \dots, p$, and middle Latin indices take values in the transverse space, *e.g.*, $i, j = p + 1, \dots, 8, 9$.

where $s = -1/2 - 2k_1 \cdot k_2$, and k^a is momentum of the open string tachyon. Expansion of this S-matrix around $s = -1/2$ is

$$A \sim \frac{1}{4} + 2 \ln(2) k_1 \cdot k_2 + O((k_1 \cdot k_2)^2) . \quad (2)$$

The first two terms of this expansion are reproduced exactly by the BSFT action[13]⁴

$$\begin{aligned} S &= -T_p \int dx \frac{\tau}{4} e^{-T^2/4} \frac{\sqrt{\pi} \Gamma((\partial T)^2 + 1)}{\Gamma((\partial T)^2 + 1/2)} , \\ &= -T_p \int dx \frac{\tau}{4} \left(1 - \frac{T^2}{4} + 2 \ln(2) (\partial T)^2 + \dots \right) , \end{aligned} \quad (3)$$

and $O((k_1 \cdot k_2)^2)$ terms are related to higher derivative terms, *e.g.*, $\partial_a \partial_b T \partial^a \partial^b T$, which are not included in (3). The expansion (2) is also completely consistent with the sigma model effective action[15]⁵. Now, expansion of S-matrix element (1) around $s = 0$ (the same limit as for the scalar case) yields

$$A \sim \frac{\pi}{8} \left(\frac{1}{2} + 2k_1 \cdot k_2 + O(s^2) \right) .$$

The first two terms here are reproduced exactly by the tachyonic DBI action[7, 8]

$$\begin{aligned} S &= -T_p \int dx \frac{\tau}{4} V(T) \sqrt{-\det(\eta_{ab} + 4\pi \partial_a T \partial_b T)} , \\ &= -T_p \int dx \frac{\tau}{4} \left(1 - \frac{\pi}{2} T^2 + 2\pi (\partial T)^2 + \dots \right) , \end{aligned} \quad (4)$$

where in the second line we have used the expansion $V(T) = 1 - \frac{\pi}{2} T^2 + O(T^4)$ at the top of the tachyon potential in this action. In this case, $O(s^2)$ terms may be related to higher derivative terms, *e.g.*, $T \partial_a \partial^a T$, which are not included in (4)⁶. Since actions (3) and (4), around the top of their potentials, are consistent with the leading terms of the S-matrix element (1) at different limits, one may expect that they are not related to each other by field redefinition. However, around the minimum of their potentials, they may be related to each other by some field redefinition.

In this paper we would like to extend the above discussion to the case of S-matrix elements of four tachyon or scalar vertex operators in the superstring theory in the presence of background B-flux. More specifically, we would like to find appropriate expansion for the S-matrix elements that correspond to the tachyonic DBI action(4). As we will see in

⁴We have used the fact that the closed string tachyon in Type 0 theory normalizes the D-brane tension as $T_p \rightarrow T_p(1 + \tau/4 + \dots)$ [14], and we have kept only linear term for the closed string tachyon.

⁵Note that the convention in [15] sets $\alpha' = 1/(2\pi)$.

⁶The higher derivative terms may have significant effect at the top of potential, however, according to the results in [1] they are not important at the minimum of tachyon potential.

the text, the background B-flux makes it easier to find the desired limit of the amplitudes. The analysis of S-matrix element of four tachyons enables us to find the coefficient of T^4 in the tachyon potential in this action.

In section 2.1, we recall the S-matrix element of four scalar vertex operators. This amplitude has three Mandelstam variables s, t, u satisfying $s + t + u = 0$, and has massless pole in s -, t - and u -channels. In this case, the DBI action correspond to the low energy expansion of the amplitude which is unique, *i.e.*, $(s, t, u) \rightarrow 0$. In section 2.2, we evaluate the S-matrix element of two scalar and two tachyon vertex operators. The amplitude here has massless pole only in t -channel. The Mandelstam variables in this case satisfy $s + t + u = -1/2$, so we expand the amplitude around $t \rightarrow 0$, $(s, u) \rightarrow -1/4$. In section 2.3, we evaluate the S-matrix element of four tachyon vertex operators. The amplitude in this case has massless pole in all s -, t -, and u -channels. The Mandelstam variables here satisfy $s + t + u = -1$. We expand the amplitude around $s \rightarrow 0$, $(t, u) \rightarrow -1/2$ to produce massless pole of field theory in s -channel, around $t \rightarrow 0$, $(s, u) \rightarrow -1/2$ to produce massless pole of field theory in t -channel, and around $u \rightarrow 0$, $(t, s) \rightarrow -1/2$ to produce the massless pole of theory in u -channel. In section 3, we compare the leading terms of the amplitudes with the non-commutative version of the tachyonic DBI action.

2 Scattering calculation

In this section, using the world-sheet conformal field theory technique, we evaluate various 4-point function of scalar and tachyon vertex operators in the presence of background B-flux. All these amplitudes have massless pole, as well as an infinite tower of massive poles. Then we find appropriate limits which reduce the amplitudes to their massless pole, and infinite number of contact terms. We begin with recalling the scattering amplitude of four massless scalar vertex operators.

2.1 Four scalars amplitude

Scattering amplitude of four vector vertex operators in superstring theory is evaluated in [16], and its low energy effective action is also studied, for example, in [17]. To find the amplitude corresponding to four scalar vertex operators, one may use the result in [16] in which the vector polarizations ζ^a are replaced by the scalar polarizations ζ^i . Since we are interested in the scattering amplitude in the presence of B-flux, one should use $G = (1/(\eta + B))_S$ as the world volume metric, and also should add an appropriate phase factor to the amplitudes in one cycling of the vertex operators with the non-commutative parameter tensor $\theta = (4\pi/(\eta + B))_A$ [18]. Adding all non-cyclic permutation of the vertex

operators, one ends up with the following amplitude:

$$A(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = A_s(\zeta_1, \zeta_2, \zeta_3, \zeta_4) + A_u(\zeta_1, \zeta_2, \zeta_3, \zeta_4) + A_t(\zeta_1, \zeta_2, \zeta_3, \zeta_4) ,$$

where A_s , A_u , and A_t are the part of the amplitude that have massless pole in s -, u - and t -channels, respectively. They are

$$\begin{aligned} A_s &= -\frac{ic}{8\pi^2 T_p} \zeta_1 \cdot \zeta_2 \zeta_3 \cdot \zeta_4 \left(\frac{\Gamma(-2s)\Gamma(1-2t)}{\Gamma(-2s-2t)} (e^{i\pi l_{12}+i\pi l_{34}} + e^{i\pi l_{14}-i\pi l_{23}}) \right. \\ &\quad + \frac{\Gamma(-2s)\Gamma(1-2u)}{\Gamma(-2s-2u)} (e^{i\pi l_{13}-i\pi l_{24}} + e^{i\pi l_{12}-i\pi l_{34}}) \\ &\quad \left. - \frac{\Gamma(1-2t)\Gamma(1-2u)}{\Gamma(1-2t-2u)} (e^{i\pi l_{14}+i\pi l_{23}} + e^{i\pi l_{13}+i\pi l_{24}}) \right) , \\ A_u &= -\frac{ic}{8\pi^2 T_p} \zeta_1 \cdot \zeta_3 \zeta_2 \cdot \zeta_4 \left(-\frac{\Gamma(1-2s)\Gamma(1-2t)}{\Gamma(1-2s-2t)} (e^{i\pi l_{12}+i\pi l_{34}} + e^{i\pi l_{14}-i\pi l_{23}}) \right. \\ &\quad + \frac{\Gamma(-2u)\Gamma(1-2s)}{\Gamma(-2u-2s)} (e^{i\pi l_{13}-i\pi l_{24}} + e^{i\pi l_{12}-i\pi l_{34}}) \\ &\quad \left. + \frac{\Gamma(-2u)\Gamma(1-2t)}{\Gamma(-2u-2t)} (e^{i\pi l_{14}+i\pi l_{23}} + e^{i\pi l_{13}+i\pi l_{24}}) \right) , \\ A_t &= -\frac{ic}{8\pi^2 T_p} \zeta_1 \cdot \zeta_4 \zeta_2 \cdot \zeta_3 \left(\frac{\Gamma(-2t)\Gamma(1-2s)}{\Gamma(-2t-2s)} (e^{i\pi l_{12}+i\pi l_{34}} + e^{i\pi l_{14}-i\pi l_{23}}) \right. \\ &\quad - \frac{\Gamma(1-2s)\Gamma(1-2u)}{\Gamma(1-2s-2u)} (e^{i\pi l_{13}-i\pi l_{24}} + e^{i\pi l_{12}-i\pi l_{34}}) \\ &\quad \left. + \frac{\Gamma(-2t)\Gamma(1-2u)}{\Gamma(-2t-2u)} (e^{i\pi l_{14}+i\pi l_{23}} + e^{i\pi l_{13}+i\pi l_{24}}) \right) , \end{aligned}$$

where $l_{ij} = k_i \cdot \theta \cdot k_j / (2\pi)$ and

$$\begin{aligned} s &= -(k_1 + k_2)^2 = -k_1^2 - k_2^2 - 2k_1 \cdot k_2 = -k_3^2 - k_4^2 - 2k_3 \cdot k_4 , \\ t &= -(k_2 + k_3)^2 = -k_2^2 - k_3^2 - 2k_2 \cdot k_3 = -k_1^2 - k_4^2 - 2k_1 \cdot k_4 , \\ u &= -(k_1 + k_3)^2 = -k_1^2 - k_3^2 - 2k_1 \cdot k_3 = -k_2^2 - k_4^2 - 2k_2 \cdot k_4 , \end{aligned} \tag{5}$$

where in the present case $k_i^2 = 0$. We have also normalized the amplitude here and in the subsequent sections by the factor $ic/(8\pi^2 T_p)$ where $c = \sqrt{-\det(\eta + B)}$. Note that each amplitude has a term which has no massless pole. They produce only contact terms upon expanding the amplitude. The amplitudes A_s , A_u and A_t are very similar, so we only analyze in some details the A_t amplitude. The low energy limit, *i.e.*, $(t, s, u) \rightarrow 0$ of the gamma functions in this amplitude are

$$\frac{\Gamma(-2t)\Gamma(1-2s)}{\Gamma(-2t-2s)} = \frac{2u}{-2t} + \frac{2\pi^2}{3}us + \dots ,$$

$$\begin{aligned}\frac{\Gamma(-2t)\Gamma(1-2u)}{\Gamma(-2t-2u)} &= \frac{2s}{-2t} + \frac{2\pi^2}{3}us + \dots, \\ \frac{\Gamma(1-2s)\Gamma(1-2u)}{\Gamma(1-2s-2u)} &= 1 - \frac{2\pi^2}{3}us + \dots,\end{aligned}$$

where we have used the on-shell condition of $s+t+u=0$, and dots above represents terms with more than four momenta. In field theory, using the non-commutative DBI action, one finds the following massless t -channel amplitude (see *e.g.*, [7] for details):

$$\begin{aligned}A'_t &= (\tilde{V}_{\phi_2\phi_3A})^a(\tilde{G}_A)_{ab}(\tilde{V}_{A\phi_1\phi_4})^b, \\ &= \left(\frac{ic}{(2\pi)^2T_p}\right)\zeta_1\cdot\zeta_4\zeta_2\cdot\zeta_3\frac{\sin(\pi l_{23})\sin(\pi l_{14})(u-s)}{t}, \\ &= \frac{-ic}{8\pi^2T_p}\zeta_1\cdot\zeta_4\zeta_2\cdot\zeta_3\left((e^{i\pi l_{12}+i\pi l_{34}}+e^{i\pi l_{14}-i\pi l_{23}})\left(\frac{2u}{-2t}-\frac{1}{2}\right)\right. \\ &\quad \left.+ (e^{i\pi l_{14}+i\pi l_{23}}-e^{i\pi l_{13}+i\pi l_{24}})\left(\frac{2s}{-2t}-\frac{1}{2}\right)\right).\end{aligned}$$

Now subtracting the field theory massless pole from the string theory scattering amplitude, one ends up with the following contact terms in the t -channel:

$$\begin{aligned}A_t - A'_t &= -\frac{ic}{8\pi^2T_p}\zeta_1\cdot\zeta_4\zeta_2\cdot\zeta_3\left((e^{i\pi l_{12}+i\pi l_{34}}+e^{i\pi l_{14}-i\pi l_{23}})\left(\frac{1}{2}+\frac{2\pi^2}{3}us+\dots\right)\right. \\ &\quad \left.+ (e^{i\pi l_{13}-i\pi l_{24}}+e^{i\pi l_{12}-i\pi l_{34}})\left(-1+\frac{2\pi^2}{3}us+\dots\right)\right. \\ &\quad \left.+ (e^{i\pi l_{14}+i\pi l_{23}}+e^{i\pi l_{13}+i\pi l_{24}})\left(\frac{1}{2}+\frac{2\pi^2}{3}us+\dots\right)\right).\end{aligned}\quad (6)$$

Doing the same analysis for the A_s and A_u , one finds the following total contact terms:

$$A_c(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = A_c^0(\zeta_1, \zeta_2, \zeta_3, \zeta_4) + A_c^4(\zeta_1, \zeta_2, \zeta_3, \zeta_4) + \sum_{n>4} A_c^n(\zeta_1, \zeta_2, \zeta_3, \zeta_4),$$

where A_c^0 contains, apart from the phase factor, contact terms with no momentum and is zero when the background B-flux vanishes,

$$\begin{aligned}A_c^0 &= \frac{-ic}{8\pi^2T_p}\left(e^{i\pi l_{12}+i\pi l_{34}}+e^{i\pi l_{14}-i\pi l_{23}}\right)\left(\frac{1}{2}\zeta_1\cdot\zeta_4\zeta_2\cdot\zeta_3+\frac{1}{2}\zeta_1\cdot\zeta_2\zeta_3\cdot\zeta_4-\zeta_1\cdot\zeta_3\zeta_2\cdot\zeta_4\right) \\ &\quad -\frac{ic}{8\pi^2T_p}\left(e^{i\pi l_{13}-i\pi l_{24}}+e^{i\pi l_{12}-i\pi l_{34}}\right)\left(-\zeta_1\cdot\zeta_4\zeta_2\cdot\zeta_3+\frac{1}{2}\zeta_1\cdot\zeta_2\zeta_3\cdot\zeta_4+\frac{1}{2}\zeta_1\cdot\zeta_3\zeta_2\cdot\zeta_4\right) \\ &\quad -\frac{ic}{8\pi^2T_p}\left(e^{i\pi l_{14}+i\pi l_{23}}+e^{i\pi l_{13}+i\pi l_{24}}\right)\left(\frac{1}{2}\zeta_1\cdot\zeta_4\zeta_2\cdot\zeta_3-\zeta_1\cdot\zeta_2\zeta_3\cdot\zeta_4+\frac{1}{2}\zeta_1\cdot\zeta_3\zeta_2\cdot\zeta_4\right).\end{aligned}\quad (7)$$

A_c^4 contains terms with four momenta which are non vanishing even in the $B=0$ limit. They are

$$A_c^4 = \frac{-ic}{6T_p}\left(e^{i\pi l_{12}+i\pi l_{34}}+e^{i\pi l_{14}-i\pi l_{23}}+e^{i\pi l_{13}-i\pi l_{24}}+e^{i\pi l_{12}-i\pi l_{34}}+e^{i\pi l_{14}+i\pi l_{23}}+e^{i\pi l_{13}+i\pi l_{24}}\right)$$

$$\begin{aligned}
& \times \left(\zeta_1 \cdot \zeta_4 \zeta_2 \cdot \zeta_3 \left((k_2 \cdot k_3)(k_1 \cdot k_4) - (k_1 \cdot k_2)(k_3 \cdot k_4) - (k_1 \cdot k_3)(k_2 \cdot k_4) \right) \right. \\
& + \zeta_1 \cdot \zeta_2 \zeta_3 \cdot \zeta_4 \left((k_1 \cdot k_2)(k_3 \cdot k_4) - (k_2 \cdot k_3)(k_1 \cdot k_4) - (k_1 \cdot k_3)(k_2 \cdot k_4) \right) \\
& \left. + \zeta_1 \cdot \zeta_3 \zeta_2 \cdot \zeta_4 \left((k_1 \cdot k_3)(k_2 \cdot k_4) - (k_1 \cdot k_2)(k_3 \cdot k_4) - (k_2 \cdot k_3)(k_1 \cdot k_2) \right) \right) . \tag{8}
\end{aligned}$$

And A_c^n with $n > 4$ contains contact terms with more than four momenta. They are related to higher derivative terms which are not included in the low energy DBI action. In writing above result we have used the condition $s + t + u = 0$ and (5). In this form as opposed to the form in (6), each momentum appears only once. This form is comparable with the DBI action which has first derivative of scalar fields. We shall compare above contact terms with the non-commutative DBI action in section 3.

2.2 Two scalars and two tachyons amplitude

The amplitude describing scattering of two tachyons to two scalars is given by the following correlation:

$$\begin{aligned}
A^{1234} & \sim \int dx_1 dx_2 dx_3 dx_4 \\
& \times \langle : V_0^T(2V \cdot k_1, x_1) : V_{-1}^X(\zeta_2, 2V \cdot k_2, x_2) : V_{-1}^X(\zeta_3, 2V \cdot k_3, x_3) : V_0^T(2V \cdot k_4, x_4) : \rangle ,
\end{aligned}$$

where we have used the doubling trick to deal with only world sheet analytic fields[19]. The matrix V is related to world volume metric and the non-commutative parameter as $V_S = G$ and $V_A = \theta/(4\pi)$ where the subscripts S and A mean symmetric and antisymmetric part of V matrix, respectively. In above equation $k_1^2 = k_4^2 = 1/4$, $k_2^2 = k_3^2 = 0$, and

$$\begin{aligned}
V_{-1}^X(\zeta, k, x) & = e^{-\phi} \zeta_i \psi^i e^{ik \cdot X} , \\
V_0^T(k, x) & = ik \cdot \psi e^{ik \cdot X} .
\end{aligned}$$

Using the appropriate world sheet propagators, one can easily evaluates all the correlators above and show that the integrand is $SL(2, \mathbb{R})$ invariant. Removing this symmetry from the integral by fixing the position of three vertex operators at $x_1 = 0$, $x_3 = 1$, and $x_4 = \infty$, one finds

$$A^{1234} \sim -4k_1 \cdot k_4 \zeta_2 \cdot \zeta_3 \int_0^1 dx x^{4k_1 \cdot k_4 - 1} (1-x)^{4k_4 \cdot k_3} e^{i\pi l_{12} + i\pi l_{34}} .$$

Adding all non-cyclic permutation of the vertex operators, one finds that the amplitude has only massless pole in t -channel, that is,

$$A(\zeta_2, \zeta_3) = A_t(\zeta_2, \zeta_3) = -\frac{ic}{8\pi^2 T_p} \zeta_2 \cdot \zeta_3 \left(\frac{\Gamma(-2t)\Gamma(1/2-2s)}{\Gamma(-1/2-2t-2s)} (e^{i\pi l_{12} + i\pi l_{34}} + e^{i\pi l_{14} - i\pi l_{23}}) \right)$$

$$\begin{aligned}
& -\frac{\Gamma(1/2-2s)\Gamma(1/2-2u)}{\Gamma(-2s-2u)}(e^{i\pi l_{13}-i\pi l_{24}} + e^{i\pi l_{12}-i\pi l_{34}}) \\
& +\frac{\Gamma(-2t)\Gamma(1/2-2u)}{\Gamma(-1/2-2t-2u)}(e^{i\pi l_{14}+i\pi l_{23}} + e^{i\pi l_{13}+i\pi l_{24}}) \Big) , \tag{9}
\end{aligned}$$

where s, t, u defined in (5), and they satisfy the on-shell condition $s + t + u = -1/2$. Now question is what is the limit that one has to take to produce the tachyonic DBI effective action? The field theory produces the following massless pole in the t -channel:

$$\begin{aligned}
A'_t &= (\tilde{V}_{\phi_2\phi_3A})^a(\tilde{G}_A)_{ab}(\tilde{V}_{AT_1T_4})^b , \\
&= \left(\frac{ic}{(2\pi)^2 T_p}\right) \zeta_2 \cdot \zeta_3 \frac{\sin(\pi l_{23}) \sin(\pi l_{14})(u-s)}{t} , \\
&= \frac{-ic}{8\pi^2 T_p} \zeta_2 \cdot \zeta_3 \left((e^{i\pi l_{12}+i\pi l_{34}} + e^{i\pi l_{14}-i\pi l_{23}}) \left(\frac{1/2+2u}{-2t} - \frac{1}{2}\right) \right. \\
&\quad \left. + (e^{i\pi l_{14}+i\pi l_{23}} - e^{i\pi l_{13}+i\pi l_{24}}) \left(\frac{1/2+2s}{-2t} - \frac{1}{2}\right) \right) . \tag{10}
\end{aligned}$$

This pole is reproduced by the string theory amplitude (9) if one takes for t the limit $t \rightarrow 0$, and for s, u the symmetric limit $(s, u) \rightarrow -1/4$. Expansion of the gamma functions in (9) in this limit are

$$\begin{aligned}
\frac{\Gamma(-2t)\Gamma(1/2-2s)}{\Gamma(-1/2-2t-2s)} &= \frac{1/2+2u}{-2t} + \frac{2\pi^2}{3}(u+1/4)(s+1/4) + \dots , \\
\frac{\Gamma(-2t)\Gamma(1/2-2u)}{\Gamma(-1/2-2t-2u)} &= \frac{1/2+2s}{-2t} + \frac{2\pi^2}{3}(u+1/4)(s+1/4) + \dots , \\
\frac{\Gamma(1/2-2s)\Gamma(1/2-2u)}{\Gamma(-2s-2u)} &= 1 - \frac{2\pi^2}{3}(u+1/4)(s+1/4) + \dots .
\end{aligned}$$

Inserting above expansion into (9), one finds that the string theory amplitude produces exactly the field theory massless pole (10) in the above mentioned limit.

To find the contact terms, one should subtract the field theory massless pole (10) from the string theory amplitude (9). Like in previous section, they can be written as

$$A_c(\zeta_2, \zeta_3) = A_c^0(\zeta_2, \zeta_3) + A_c^4(\zeta_2, \zeta_3) + \sum_{n>4} A_c^n(\zeta_2, \zeta_3) , \tag{11}$$

where A_c^0 includes the contact terms that have no momentum and is zero when the background field vanishes,

$$\begin{aligned}
A_c^0 &= -\frac{ic}{8\pi^2 T_p} \zeta_2 \cdot \zeta_3 \left(\frac{1}{2} (e^{i\pi l_{12}+i\pi l_{34}} + e^{i\pi l_{14}-i\pi l_{23}}) \right. \\
&\quad \left. - (e^{i\pi l_{13}-i\pi l_{24}} + e^{i\pi l_{12}-i\pi l_{34}}) + \frac{1}{2} (e^{i\pi l_{14}+i\pi l_{23}} + e^{i\pi l_{13}+i\pi l_{24}}) \right) . \tag{12}
\end{aligned}$$

Using the fact that $u+1/4 = -2k_1 \cdot k_3$ and $s+1/4 = -2k_1 \cdot k_2$, one finds that A_c^4 contact terms are proportional to $(k_1 \cdot k_2)(k_1 \cdot k_3)$. However, the contact terms in this form is not appropriate to compare with an action which has only first derivative terms, since k_1 appears with power two and k_4 does not appear at all. They can be rewritten in the suitable form using the on-shell condition $s + t + u = -1/2$, and (5),

$$A_c^4 = \frac{-ic}{6T_p} \left(e^{i\pi l_{12} + i\pi l_{34}} + e^{i\pi l_{14} - i\pi l_{23}} + e^{i\pi l_{13} - i\pi l_{24}} + e^{i\pi l_{12} - i\pi l_{34}} + e^{i\pi l_{14} + i\pi l_{23}} + e^{i\pi l_{13} + i\pi l_{24}} \right) \\ \times \zeta_2 \cdot \zeta_3 \left((k_2 \cdot k_3)(k_1 \cdot k_4) - (k_1 \cdot k_2)(k_3 \cdot k_4) - (k_1 \cdot k_3)(k_2 \cdot k_4) + \frac{1}{4}(k_2 \cdot k_3) \right). \quad (13)$$

In this form each momentum appears at most once which is in appropriate form for comparing with the tachyonic DBI action which includes at most the first derivative of scalars and tachyon. The contact terms A_c^n with $n > 4$ in (11) contains all other terms. One may expect that they are related to higher derivative terms that are not included in the tachyonic DBI action. We find a justification for this in section 3.

2.3 Four tachyons amplitude

The amplitude describing scattering of two tachyons to two tachyons is given by the following correlation:

$$A^{1234} \sim \int dx_1 dx_2 dx_3 dx_4 \\ \times \langle : V_{-1}^T(2V \cdot k_1, x_1) : V_{-1}^T(2V \cdot k_2, x_2) : V_0^T(2V \cdot k_3, x_3) : V_0^T(2V \cdot k_4, x_4) : \rangle,$$

where $k_i^2 = 1/4$, and

$$V_{-1}^T(k, x) = e^{-\phi} e^{ik \cdot X}, \\ V_0^T(k, x) = ik \cdot \psi e^{ik \cdot X}.$$

Here again it is easy to evaluate all the correlators in the scattering amplitude above and to show the integrand is SL(2,R) invariant. Removing this symmetry by fixing $x_1 = 0, x_3 = 1$, and $x_4 = \infty$, one finds

$$A^{1234} \sim -4k_3 \cdot k_4 \int_0^1 dx x^{4k_1 \cdot k_2 - 1} (1-x)^{4k_2 \cdot k_3} e^{i\pi l_{12} + i\pi l_{34}}.$$

Adding all non-cyclic permutations of the vertex operators, one finds that the whole amplitude has massless pole in all s -, t - and u -channels, that is,

$$A = A_s + A_u + A_t.$$

However, in this case the amplitudes in all channels are identical *i.e.*, $A_s = A_t = A_u$, and

$$\begin{aligned}
A_t = & -\frac{ic}{8\pi^2 T_p} \left(\frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-1-2s-2t)} (e^{i\pi l_{12}+i\pi l_{34}} + e^{i\pi l_{14}-i\pi l_{23}}) \right. \\
& - \frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} (e^{i\pi l_{13}-i\pi l_{24}} + e^{i\pi l_{12}-i\pi l_{34}}) \\
& \left. + \frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)} (e^{i\pi l_{14}+i\pi l_{23}} + e^{i\pi l_{13}+i\pi l_{24}}) \right), \tag{14}
\end{aligned}$$

where s, t, u satisfy the on-shell condition $s + t + u = -1$. Here again to produce the t -channel massless pole in field theory one has to take the limit $t \rightarrow 0$ for t , and the symmetric limit $(s, u) \rightarrow -1/2$ for s, u . The gamma functions in A_t in this limit are

$$\begin{aligned}
\frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-1-2t-2s)} &= \frac{1+2u}{-2t} + \frac{2\pi^2}{3}(u+1/2)(s+1/2) + \dots, \\
\frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)} &= \frac{1+2s}{-2t} + \frac{2\pi^2}{3}(u+1/2)(s+1/2) + \dots, \\
\frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} &= 1 - \frac{2\pi^2}{3}(u+1/2)(s+1/2) + \dots.
\end{aligned}$$

It is important to note that even though $\Gamma(-2s)$ and $\Gamma(-2u)$ in above equations have massless pole in s - and u -channels, respectively, they produce only contact terms in the limit $(s, u) \rightarrow -1/2$ taken for A_t . The field theory result for massless t -channel pole is

$$\begin{aligned}
A'_t &= (\tilde{V}_{T_2 T_3 A})^a (\tilde{G}_A)_{ab} (\tilde{V}_{AT_1 T_4})^b, \\
&= \left(\frac{ic}{(2\pi)^2 T_p} \right) \frac{\sin(\pi l_{23}) \sin(\pi l_{14})(u-s)}{t}, \\
&= \frac{-ic}{8\pi^2 T_p} \left((e^{i\pi l_{12}+i\pi l_{34}} + e^{i\pi l_{14}-i\pi l_{23}}) \left(\frac{1+2u}{-2t} - \frac{1}{2} \right) \right. \\
&\quad \left. + (e^{i\pi l_{14}+i\pi l_{23}} - e^{i\pi l_{13}+i\pi l_{24}}) \left(\frac{1+2s}{-2t} - \frac{1}{2} \right) \right).
\end{aligned}$$

Subtracting the above field theory amplitude from the string theory amplitude(14), one finds

$$\begin{aligned}
A_t - A'_t = & -\frac{ic}{8\pi^2 T_p} \left((e^{i\pi l_{12}+i\pi l_{34}} + e^{i\pi l_{14}-i\pi l_{23}}) \left(\frac{1}{2} + \frac{2\pi^2}{3}(u+1/2)(s+1/2) + \dots \right) \right. \\
& + (e^{i\pi l_{13}-i\pi l_{24}} + e^{i\pi l_{12}-i\pi l_{34}}) \left(-1 + \frac{2\pi^2}{3}(u+1/2)(s+1/2) + \dots \right) \\
& \left. + (e^{i\pi l_{14}+i\pi l_{23}} + e^{i\pi l_{13}+i\pi l_{24}}) \left(\frac{1}{2} + \frac{2\pi^2}{3}(u+1/2)(s+1/2) + \dots \right) \right).
\end{aligned}$$

For A_s amplitude, one has to take the limit $s \rightarrow 0$, $(t, u) \rightarrow -1/2$, and similarly for A_u one has to take the limit $u \rightarrow 0$, $(s, t) \rightarrow -1/2$ to produce the field theory massless pole in

u -channel . Adding the contact terms in all channels, one finds the following total contact terms:

$$A_c = A_c^0 + A_c^4 + \sum_{n>4} A_c^n ,$$

where in this case $A_c^0 = 0$, as expected (see section 3), A_c^4 which has four momenta can be written in the following form:

$$A_c^4 = -\frac{i c}{6 T_p} \left(e^{i\pi l_{12} + i\pi l_{34}} + e^{i\pi l_{14} - i\pi l_{23}} + e^{i\pi l_{13} - i\pi l_{24}} + e^{i\pi l_{12} - i\pi l_{34}} + e^{i\pi l_{14} + i\pi l_{23}} + e^{i\pi l_{13} + i\pi l_{24}} \right) \\ \times \left(-(k_2 \cdot k_3)(k_1 \cdot k_4) - (k_1 \cdot k_2)(k_3 \cdot k_4) - (k_1 \cdot k_3)(k_2 \cdot k_4) + \frac{1}{16} \right) , \quad (15)$$

where we have used the on-shell condition $s + t + u = -1$ and (5) to write the momenta in this amplitude in the form that each momentum appears at most once. And A_c^n with $n > 4$ contains all other infinite contact terms. They may be related to higher derivative terms that are not included in the tachyonic DBI action. In next section we shall compare A_c^0 's and A_c^4 's with the tachyonic DBI action.

3 Effective action

In [7], the S-matrix element of two tachyons and one massless closed string $NSNS$ state in the presence of background B-flux was examined in details in order to find a tachyonic effective action. The string theory amplitude in this simple case has only one Mandelstam variable, s , and it also has massless pole as well as infinite number of massive poles. The only limit that reduces the amplitude to its massless pole and infinite number of contact terms is the limit $s \rightarrow 0$. In this limit, the contact terms of the S-matrix element contain $*$ ' which indicates that the effective action, like the low energy non-commutative DBI action, should also have Wilson line [12, 20]. Then it was shown that the massless pole and the leading contact term of the amplitude are fully consistent with the non-commutative version of the following tachyonic DBI action[7, 8]⁷ :

$$S = -T_p \int d^{p+1} \sigma V(T) e^{-\Phi} \sqrt{-\det(P[g_{ab} + b_{ab}] + 2\pi\alpha' F_{ab} + 2\pi\alpha' \partial_a T \partial_b T)} , \quad (16)$$

where $V(T) = 1 - \frac{\pi}{2} T^2 + O(T^4)$, g_{ab} is flat space metric η_{ab} plus its graviton fluctuation. Here b_{ab} , Φ , A_a and T are the antisymmetric Kalb-Ramond tensor, dilaton, gauge field and the tachyon fluctuations, respectively. In above action $P[\dots]$ is also the pull-back of the closed string fields. For example, $P[\eta_{ab}] = \eta_{ab} + \partial_a X^i \partial_b X_i$ in the static gauge.

⁷We explicitly restore α' in this section.

The non-abelian extension of this action is[7]

$$S = -T_p \int d^{p+1} \sigma \text{Tr} \left(V(T) \sqrt{\det(Q^i_j)} \right. \\ \left. \times e^{-\Phi} \sqrt{-\det(P[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij}E_{jb}] + 2\pi\alpha' F_{ab} + T_{ab})} \right) , \quad (17)$$

where $E_{\mu\nu} = g_{\mu\nu} + b_{\mu\nu}$. The indices in this action are raised and lowered by E^{ij} and E_{ij} , respectively. The matrices Q^i_j and T_{ab} are

$$Q^i_j = \delta^i_j - \frac{i}{2\pi\alpha'} [X^i, X^k] E_{kj} - \frac{1}{2\pi\alpha'} [X^i, T] [X^k, T] E_{kj} , \\ T_{ab} = 2\pi\alpha' D_a T D_b T + D_a T [X^i, T] (Q^{-1})_{ij} [X^j, T] D_b T \\ + i E_{ai} (Q^{-1})^i_j [X^j, T] D_b T + i D_a T [X^i, T] (Q^{-1})_i^j E_{jb} \\ + i D_a X^i (Q^{-1})_{ij} [X^j, T] D_b T - i D_a T [X^i, T] (Q^{-1})_{ij} D_b X^j .$$

The trace in the action (17) should be completely symmetric between all non-abelian expression of the form F_{ab} , $D_a X^i$, $[X^i, X^j]$, $D_a T$, $[X^i, T]$, individual T of the tachyon potential and individual X^i of the Taylor expansion of the closed string fields in the action[21]. All world volume derivatives in this action are covariant derivatives.

The abelian non-commutative action when there is a background B-flux with non-vanishing component only in the world volume directions *i.e.*, $B_{ab} \neq 0$, and when the closed string fields are constant *i.e.*, no closed string quantum fluctuation, is the same as the non-abelian action (17) in which the world volume metric should be replaced by $G = (1/(\eta+B))_S$, the ordinary multiplication should be replaced by $*$ -product with non-commutative parameter $\theta = (2\pi\alpha'/(\eta+B))_A$, and $T_p \rightarrow cT_p$ [18]. Note that the symmetrized trace in non-abelian case transforms to symmetrized $*$ -product in the abelian non-commutative action.

Now it is straightforward to expand (17) to find different couplings in this action. The couplings between four non-commutative scalar fields is

$$\mathcal{L}(X, X, X, X) = -cT_p \left(\frac{1}{16\pi^2\alpha'^2} [X^i, X^k] [X_k, X_i] \right. \\ \left. - \frac{1}{4} (\partial_a X^i \partial_b X_i) (\partial^b X_j \partial^a X^j) + \frac{1}{8} (\partial_a X^i \partial^a X_i)^2 \right) , \quad (18)$$

where the multiplication is the symmetrized $*$ -product. Using the rescaling $X^i \rightarrow \frac{1}{\sqrt{T_p}} X^i$, it is easy to verify that the coupling in the first line produce the contact terms (7) in momentum space. The symmetrized $*$ -product for the coupling in the first line above is trivial, whereas it has nontrivial effect on the terms in the second line. Using this, one can show that they produce exactly the contact terms in (8). All these confirm the consistency between string theory scattering amplitude at low energy and the low energy non-commutative DBI action with symmetrized $*$ -product. Now let's examine the coupling

between two scalars and two tachyons. Using the symmetrized product, one finds only the following couplings:

$$\begin{aligned} \mathcal{L}(X, X, T, T) = & -cT_p \left(\frac{1}{4\pi\alpha'} [X^i, T][X_i, T] - \frac{\pi}{4} T^2 (\partial_a X^i \partial^a X_i) \right. \\ & \left. - \pi\alpha' (\partial_a X^i \partial_b X_i) (\partial^b T \partial^a T) + \frac{\pi\alpha'}{2} (\partial_a X^i \partial_a X_i) (\partial_b T \partial^b T) \right), \end{aligned} \quad (19)$$

where again the multiplication rule is the symmetrized $*$ -product, and the world volume indices are raised by G^{ab} . Now using the rescaling $T \rightarrow \frac{1}{\sqrt{2\pi\alpha'T_p}} T$, one can show that the first term in the first line above, reproduce the contact terms in (12), the second term in the first line above produces the last contact term in (13), and the other couplings in the second line above produce the contact terms in (13) which have four momenta. This confirms that the coupling extracting from the tachyonic DBI action are consistent with the string theory scattering amplitude at the limit $t \rightarrow 0$, $(s, u) \rightarrow -1/4$. This also justifies the ignoring of the A_c^n terms with $n > 4$, *i.e.*, they are related to higher derivative terms that are not included in the tachyonic DBI action (17). Finally, the coupling of four tachyons extracted from (17) is

$$\mathcal{L}(T, T, T, T) = -cT_p \left(\beta T^4 - \frac{\pi^2\alpha'}{2} T^2 (\partial_a T \partial^a T) - \frac{\pi^2\alpha'^2}{2} (\partial_a T \partial^a T)^2 \right), \quad (20)$$

where again the product is the symmetrized $*$ -product and the constant β is the yet unknown coefficient of T^4 in the tachyon potential. Here once again one can observe that the terms with four derivatives reproduce exactly the four momentum contact terms in (15), and the rest produce the contact terms with no momentum in (15) provided $\beta = \pi^2/8$.

The tachyon potential expanded around its maximum, *i.e.*, around $T_{max} = 0$, is then

$$V(T) = 1 - \frac{\pi}{2} T^2 + \frac{\pi^2}{8} T^4 + O(T^6). \quad (21)$$

The coefficient of T^4 is consistent with Sen's conjecture that the tachyon potential should have a minimum as well[22]. In fact if we ignore the $O(T^6)$ terms in the potential, $V(T)$ has minimum at $T_{min} = \sqrt{2/\pi}$, and the minimum of the potential is $V(T_{min}) = 0.5$ which is different from the actual value of zero. This means that the higher order terms $O(T^6)$ should be included in the potential. In fact Sen has shown that the actual tachyon potential in the action (16) has minimum at $T_{min} \rightarrow \infty$, and the behavior of the potential around the minimum should be like $e^{-\sqrt{\pi}T}$ [23]⁸. One may try to find a tachyon potential with this behavior at $T_{min} \rightarrow \infty$, and expansion (21) at $T_{max} = 0$. For example the following function

$$V(T) = \frac{1}{2 \cosh(\sqrt{\pi}T)} \left(1 + \frac{1}{1 + \pi^2 T^4/6} \right),$$

⁸Note that the convention in [23] sets $\alpha' = 1$.

has the correct behaviors at minimum and maximum of the potential. It would be interesting to extend the method used in this paper to find the $O(T^6)$ terms, and consequently to find the actual form of the tachyon potential in the tachyonic DBI action(16). It would be also interesting to apply this method to the bosonic string theory S-matrix elements which have both tachyonic and massless poles.

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